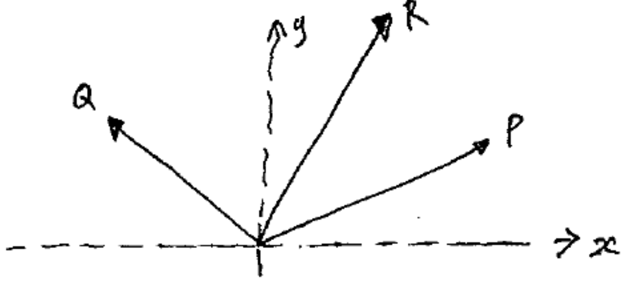
M1 May 2007 Solutions (OCR 47

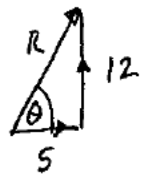
1.



i) Resultant of P and Q, $R = \begin{pmatrix} 14 \\ 5 \end{pmatrix} + \begin{pmatrix} -9 \\ 7 \end{pmatrix} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$

Components of Resultant, R, in the x- and y- directions are $\underline{\underline{5N \text{ and } 12N}}$ respectively.

ii)



Magnitude of $R = \sqrt{5^2+12^2} = \underline{\underline{13N}}$

Angle with positive x axis, $\theta = \tan^{-1}\left(\frac{12}{5}\right) = \underline{\underline{67.4^\circ}}$

2. i) Particle [redacted] starts to return towards A when the velocity becomes negative. As deceleration is constant between $12ms^{-1}$ & $-12ms^{-1}$, and 250 s & 290 s, the particle distance from A is greatest at $t = 250 + \left(\frac{290-250}{2}\right) = \underline{\underline{270s}}$

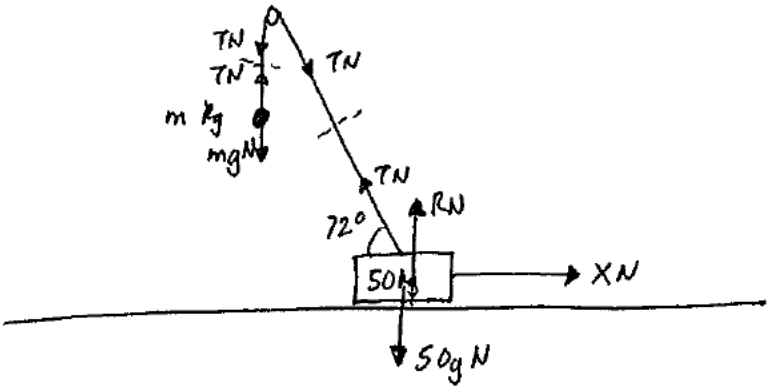
ii) Displacement of particle from A when $t = 290s$

$$= \frac{1}{2}(a+b_1)h - \frac{1}{2}b_2h = \frac{1}{2}(210+270)\times 12 - \frac{1}{2}\times 20\times 12 = 2880 - 120$$

$$= \underline{\underline{2760 \text{ m}}}$$

iii) Total distance travelled $= 2880 + 120 = \underline{\underline{3000 \text{ m}}}$

3.



i) Resolving vertically :- $T\sin 72° + R = 50g$

ii) At point of lifting off ground, $R = 0$

$\therefore \; T\sin 72° = 50g$

$$T = \frac{50 \times 9.8}{\sin 72°} = 515\,N$$

Resolving vertically at object:-

$$T = mg$$

Substituting for T :-

$$515 = mg$$

$$m = \frac{515}{9.8} = 52.6\ kg$$

iii) Resolving horizontally :- $T\cos 72° = X$

$$X = 515\cos 72° = 159\,N$$

4. i) $m_A u_A + m_B u_B = m_A v_A + m_B v_B$

$0.18 \times 2 - m \times 3 = 0$

$3m = 0.18 \times 2$

$m = (0.18 \times 2)/3 = 0.12\ kg$

ii) a) $0.18 \times 2 - m \times 3 = -0.18 \times 1.5 + m \times 1.5$

$0.18 \times 2 + 0.18 \times 1.5 = 1.5m + 3m$

$(0.18 \times 3.5)/4.5 = m = 0.14\ kg$

b) $0.18 \times 2 - m \times 3 = -0.18 \times 1.5 - m \times 1.5$

$0.18 \times 2 + 0.18 \times 1.5 = 3m - 1.5m$

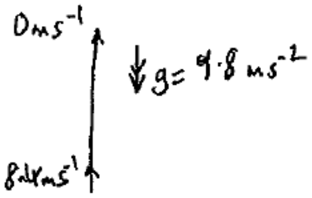
$(0.18 \times 3.5)/1.5 = m = 0.42\ kg$

c) $0.18 \times 2 - m \times 3 = 0.18 \times 1.5 + m \times 1.5$

$0.18 \times 2 - 0.18 \times 1.5 = 3m + 1.5m$

$(0.18 \times 0.5)/4.5 = m = 0.02\ kg$

5.

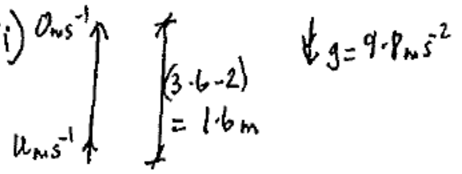


i) $v^2 = u^2 + 2as$ $\quad v = 0\,ms^{-1}$; $u = 8.4\,ms^{-1}$

$$s = \frac{v^2 - u^2}{2a}$$

$$s = \frac{0^2 - 8.4^2}{2 \times -9.8} = \underline{\underline{3.6\,m}}$$

ii)



ii) $v^2 = u^2 + 2as$

$$u = \sqrt{v^2 - 2as}$$

$$u = \sqrt{0^2 - 2 \times -9.8 \times 1.6} = \underline{\underline{5.6\,ms^{-1}}}$$

iii) Using $v = u + at$

For P, time to greatest height, $t = \frac{v-u}{a} = \frac{0 - 8.4}{-9.8} = 0.8571\,s$

For Q, time to greatest height, $t = \frac{v-u}{a} = \frac{0 - 5.6}{-9.8} = 0.5714\,s$

As acceleration downwards is the same as deceleration upwards, P & Q will be the same height at a time halfway between P & Q at the greatest height

$= (0.8571 + 0.5714)/2 = 0.71425\,s$

Speed of P at $0.71425\,s$, $v = 8.4 - 9.8 \times 0.71425 = \underline{\underline{1.4\,ms^{-1}\text{ upwards.}}}$

Speed of Q at $0.71425\,s$, $v = 0 + 9.8 \times (0.71425 - 0.5714) = \underline{\underline{1.4\,ms^{-1}\text{ downwards.}}}$

6. $s = 0.001t^4 - 0.04t^3 + 0.6t^2$

$$v = \frac{ds}{dt} = 0.004t^3 - 0.12t^2 + 1.2t$$

i) When $t = 10$, $v = 0.004 \times 10^3 - 0.12 \times 10^2 + 1.2 \times 10$

$$= 4 - 12 + 12 = \underline{\underline{4\,ms^{-1}}}$$

ii) $a = 0.8 - 0.08t$

$$v = \int (0.8 - 0.08t)\,dt = 0.8t - 0.04t^2 + c$$

When $t = 10s$, $v = 4\,ms^{-1}$ $\Rightarrow$ $4 = 0.8 \times 10 - 0.04 \times 10^2 + c$ $\Rightarrow c = 4 - 8 + 4 = 0$

When $t = 20s$, $v = 0.8 \times 20 - 0.04 \times 20^2 = 16 - 16 = \underline{\underline{0\,ms^{-1}}}$

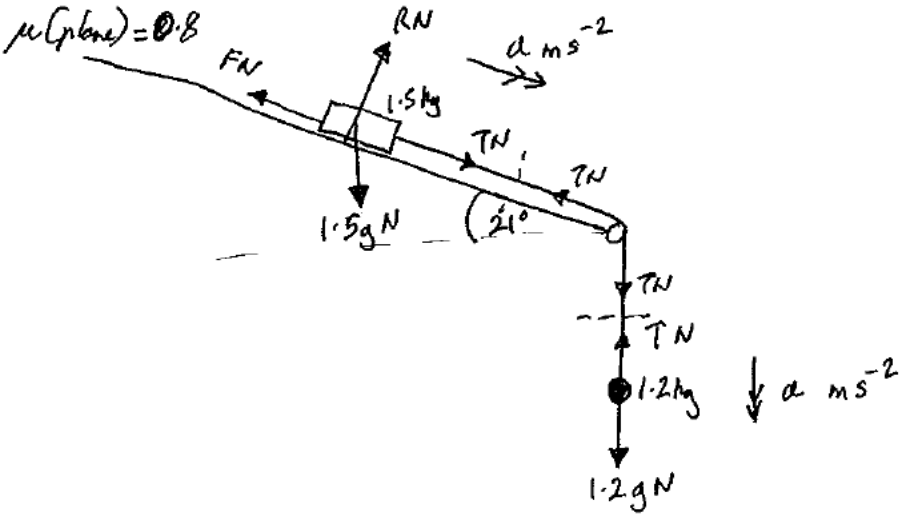
iii) $v = 0.8t - 0.04t^2$

$s = \int (0.8t - 0.04t^2)\,dt = 0.4t^2 - 0.01\dot{3}t^3 + k$

From $s = 0.001t^4 - 0.04t^3 + 0.6t^2$, when $t = 10s$, $s = 30\,m$ $\Rightarrow 30 = 0.4 \times 10^2 - 0.01\dot{3} \times 10^3 + k \Rightarrow k = 3.\dot{3}$

When $t = 20s$, $s = 0.4 \times 20^2 - 0.01\dot{3} \times 20^3 + 3.\dot{3} = \underline{\underline{56.7\,m}}$

7.



i) As the block is sliding, $F = F_{max} = \mu R = 0.8R$ ①

Resolve ⊥ plane :- $R = 1.5g\cos 21°$ ②

Substitute ② in ① :- $F = 0.8 \times 1.5 \times 9.8 \cos 21° = \underline{\underline{10.98N}}$

ii) Resolving // plane :- $T + 1.5g\sin 21° - 10.98 = 1.5a$ ①

Resolve vertically at object :- $1.2g - T = 1.2a$ ②

iii) Substitute ② in ① in (ii) above :-

$$1.2g - 1.2a + 1.5g\sin 21° - 10.98 = 1.5a$$
$$1.2 \times 9.8 + 1.5 \times 9.8 \sin 21° - 10.98 = 1.5a + 1.2a$$
$$1.2 \times 9.8 + 1.5 \times 9.8 \sin 21° - 10.98 = 2.7a$$
$$a = \underline{\underline{2.24\ ms^{-2}}}$$

iv) a) $0\,ms^{-1}$, $v\,ms^{-1}$, 2m, $2.24\,ms^{-2}$

$$v^2 = u^2 + 2as$$
$$v = \sqrt{u^2 + 2as}$$
$$= \sqrt{0^2 + 2 \times 2.24 \times 2}$$
$$= \underline{\underline{2.99\,ms^{-1}}}$$

b) When object hits ground, $T = 0$ ∴ // plane $1.5g\sin 21° - 10.98 = 1.5a$ ∴ $a = -3.81\,ms^{-2}$

At 0.8m from pulley, speed of block = $2.99\,ms^{-1}$

∴ $v^2 = u^2 + 2as$, $v = \sqrt{u^2 + 2as}$

$$= \sqrt{2.99^2 + 2 \times (-3.81) \times 0.8}$$
$$= \underline{\underline{1.69\,ms^{-1}}}$$